

# Active Power Filter: Optimal Current Control

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**Abstract** – The command of the transistors of an active power filter is studied and analysed. It has been proposed a control method based in the optimal control theory. Three different situations are compared, and have been achieved the best results with the current optimal control.

**Index terms** – harmonic distortion, active power filter, optimal control, inverter, converter

## I. Introduction

The power quality is actually a matter of great discussion, being a consequence of the general use of electronic components, polluters of the electrical systems. The current harmonics is one cause of such lowering of power quality, and thus a problem that urge to solve. The easiest way is to eliminate the harmonics of the load current with another current produced in a shunt active power filter, resulting in a sinusoidal waveform input current (Fig. 1).

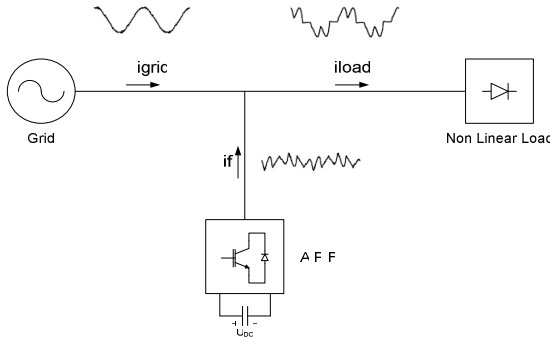


Fig. 1: general scheme of the active power filter

The grid acts as an ideal voltage source, and the load is composed by power conversion devices, mainly full-wave rectifiers.

## II. Mathematical Model

The A.P.F. circuit consists of an inverter, with two branches of IGBT transistors (cf. Fig. 2). It acts as a current source, generating  $i_f$  which compensates  $i_{load}$  in close loop; the system receives a sinusoidal waveform current from the source.

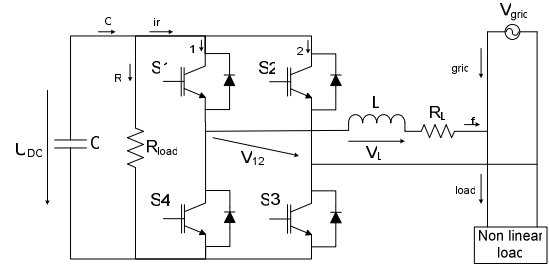


Fig. 2: Circuit of the shunt A.P.F.

The variable  $\gamma$  expresses the state of the IGBTs in the circuit, as represented in

$$\gamma = \begin{cases} -1, & \text{se } S2 \text{ ON, } S4 \text{ ON} \\ 0, & \text{se } S1 \text{ ON, } S2 \text{ ON ou } S3 \text{ ON, } S4 \text{ ON} \\ 1, & \text{se } S1 \text{ ON, } S3 \text{ ON} \end{cases} \quad (1)$$

If we apply the Kirchoff laws to the circuit, and calculate in order to the variables to be controlled - the current injected by the filter,  $i_f$ , and the voltage in the capacitor,  $U_{DC}$  - we have the dynamic model of the circuit (Eqs (2) to (5)).

$$\frac{dU_{DC}(t)}{dt} = -\frac{1}{C} \left[ \gamma i_f(t) + \frac{U_{DC}(t)}{R_C} \right] \quad (2)$$

$$\frac{di_f(t)}{dt} = \frac{1}{L} \left[ \gamma U_{DC}(t) - R_L i_f(t) - V_s(t) \right] \quad (3)$$

$$\gamma i_f(t) = i_c(t) - i_{Rc}(t) \quad (4)$$

$$i_{grid} = i_{load} - i_f \quad (5)$$

## III. Command and Control

The control is based in two variables:  $i_f$  and  $U_{DC}$ . So, there are two types of control, current control and voltage control. The first one is divided in two analyses, the conventional type (hysteresis comparing) and the optimal control type. The voltage control is necessary to generate the reference current,  $i_{grid}^*$ , which is compared with the sampled grid current,  $i_{grid}$ . The error is treated in different ways in the two types of current control, and generates the

variable  $\gamma$ . The block diagram of the A.P.F. control can be seen in Fig. 3.

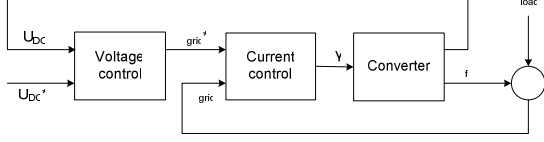


Fig. 3: Block diagram of the A.P.F. control

The current control based on sliding mode uses the error resultant of comparing  $i_{grid}$  with  $i_{grid}^*$  to generate  $\gamma$ . Its block diagram can be seen in Fig. 4.

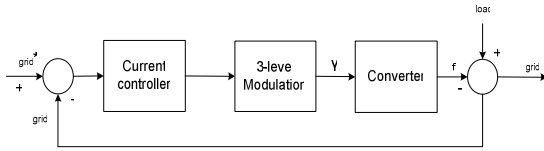


Fig. 4: Block diagram of current control

Let be considered the dynamic model of the A.P.F. ((2), (3), (4) and (5)), and the error that must be eliminated,

$$e = i_{rede}^* - i_{rede} \quad (6)$$

The three different functioning zones in (1) can be defined as follows:

$$e < 0 \Leftrightarrow i_{rede}^* < i_{rede} \Leftrightarrow i_{rede}^* < i_{carga} - i_f \Rightarrow i_f \nearrow \Rightarrow \frac{di_f}{dt} > 0 \Rightarrow \gamma = 1 \quad (7)$$

$$e = 0 \Leftrightarrow i_{rede}^* = i_{rede} \Leftrightarrow i_{rede}^* = i_{carga} - i_f \Rightarrow i_f \rightarrow \Rightarrow \frac{di_f}{dt} = 0 \Rightarrow \gamma = 0 \quad (8)$$

$$e > 0 \Leftrightarrow i_{rede}^* > i_{rede} \Leftrightarrow i_{rede}^* > i_{carga} - i_f \Rightarrow i_f \searrow \Rightarrow \frac{di_f}{dt} < 0 \Rightarrow \gamma = -1 \quad (9)$$

So, the control variable  $\gamma$  can be defined as follows:

$$\gamma = \begin{cases} 1, & e < 0 \\ 0, & e = 0 \\ -1, & e > 0 \end{cases} \quad (10)$$

In the sliding mode control, the error stability must be respected,

$$e \frac{de}{dt} < 0 \quad (11)$$

It's defined an interval  $-\Delta_e < e < \Delta_e$ , where the limits  $\Delta_e$  are related with the maximum commutation frequency of the semiconductors. The error respects the following condition:

$$e \leq 2\Delta_e \quad (12)$$

The modulation in 3 levels requires two hysteretic comparators (cf. Fig. 5), satisfying (11) and indicating which signal has the reference current (positive or negative).

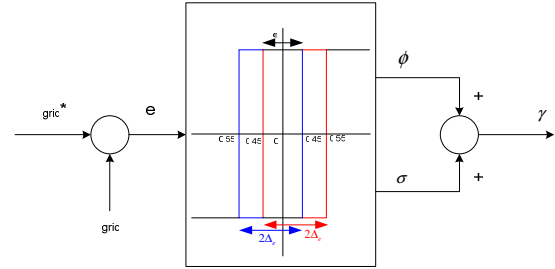


Fig. 5: hysteretic blocks

Given the flow of energy between A.P.F., grid and load, the variations of  $U_{DC}$  are used to generate the reference current. This voltage is compared with its reference value,  $U_{DCref}$ , and the resulting error is the input of the voltage control loop, Fig. 6.

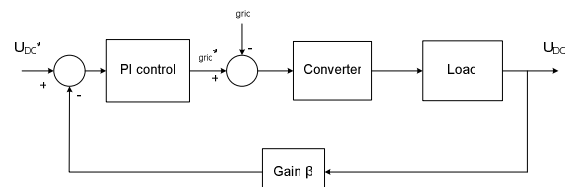


Fig. 6: block diagram of voltage control

The voltage control has a PI controller [1], whose proportional component gives the system a quick response and whose integral component allows the elimination of the error when the system is stationary. The PI controller has a transfer function given by

$$C(s) = \frac{T_z s + 1}{T_p s} \quad (13)$$

where the time constants  $T_z$  and  $T_p$  are

$$\begin{aligned} T_z &= C.R_C \\ T_p &= 4\beta GR_C \xi^2 T_d \end{aligned} \quad (14)$$

where  $\beta$  is the close loop gain,  $G$  is the converter gain and  $T_d$  is the system response delay.

The current optimal control, whose block diagram can be seen in Fig. 7, is an application of the optimal control theory. Given the non linearity of the studied system, its optimal control must have a sufficient condition and not only a necessary condition: the current optimal control is based in the Hamilton Equation theory.

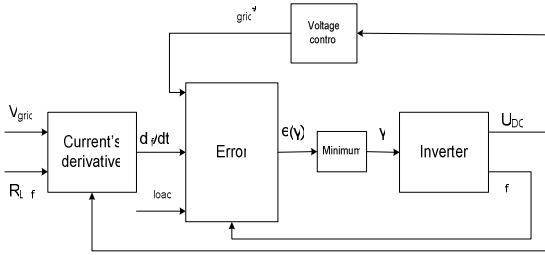


Fig. 7: block diagram of the optimal current control

The objective is to predict the current  $i_{grid}$ ; the Fourier series calculus results in (15)

$$i_{grid\_t+1} \approx i_{grid} + \Delta t \cdot \frac{di_{grid}}{dt} \quad (15)$$

and the error is given by

$$e = i_{grid\_t+1}^* - i_{grid\_t+1} \quad (16)$$

Applying the system main equation, (5), results in

$$i_{grid\_t+1} = i_{load\_t+1} - i_{f\_t+1} \quad (17)$$

The load current is almost constant in short periods comparing to the other currents in the system, and so it is considered a constant in time. Also, the grid current is going to be proportional to filter current, and so

$$\begin{aligned} i_{load\_t+1} &\approx i_{load} \\ i_{f\_t+1} &\approx i_{f\_t} + \Delta t \frac{di_f}{dt} \end{aligned} \quad (18)$$

Replacing it in (17):

$$i_{grid\_t+1} = i_{load\_t} - \left( i_{f\_t} + \Delta t \frac{di_f}{dt} \right) \quad (19)$$

The error must be minimized, and so

$$i_{grid\_t+1}^* = i_{load\_t} - \left( i_{f\_t} + \Delta t \frac{di_f}{dt} \right) \quad (20)$$

To avoid the differential calculus in *Simulink*, it's used (3) in next steps. The error is computed to the three different values of  $\gamma$ , given in (10):

$$\begin{aligned} \frac{di_f(t)}{dt}_{\gamma=-1} &= \frac{1}{L} \left[ -U_{DC}(t) - R_L i_f(t) - V_S(t) \right] \\ \frac{di_f(t)}{dt}_{\gamma=0} &= \frac{1}{L} \left[ -R_L i_f(t) - V_S(t) \right] \\ \frac{di_f(t)}{dt}_{\gamma=1} &= \frac{1}{L} \left[ U_{DC}(t) - R_L i_f(t) - V_S(t) \right] \end{aligned} \quad (21)$$

Replacing it in (20), results in

$$\begin{aligned} i_{grid\_t+1}(\gamma=-1) &= i_{load\_t} - \left( i_{f\_t} + \Delta t \frac{di_f}{dt}_{\gamma=-1} \right) \\ i_{grid\_t+1}(\gamma=0) &= i_{load\_t} - \left( i_{f\_t} + \Delta t \frac{di_f}{dt}_{\gamma=0} \right) \\ i_{grid\_t+1}(\gamma=1) &= i_{load\_t} - \left( i_{f\_t} + \Delta t \frac{di_f}{dt}_{\gamma=1} \right) \end{aligned} \quad (22)$$

Replacing in (16), it's computed the error possible values:

$$\begin{aligned} e(\gamma=-1) &= \left| i_{grid\_t+1} - i_{grid\_t+1}(\gamma=-1) \right| \\ e(\gamma=0) &= \left| i_{grid\_t+1} - i_{grid\_t+1}(\gamma=0) \right| \\ e(\gamma=1) &= \left| i_{grid\_t+1} - i_{grid\_t+1}(\gamma=1) \right| \end{aligned} \quad (23)$$

Among the computed values, it's calculated the minimum:

$$e = \min \{ e(\gamma=-1), e(\gamma=0), e(\gamma=1) \} \quad (24)$$

Thus the control system attributes to  $\gamma$  the value respecting to the minimum error.

#### IV. Simulation of A.P.F.

There are three fundamental simulations: no A.P.F., A.P.F. with sliding mode control and A.P.F. with optimal current control. There are common characteristics among them: the grid source, supplying an input voltage to the circuit,  $V_S = \sqrt{2}V_{Sef} \sin(2\pi f_{grid}t)$ ,  $V_{Set} = 230$  V,  $f_{grid} = 50$  Hz; and the RC load, typical of modern electronic components.

In the first simulation, given the absence of filter, the grid current is equal to the load current. In Figs. 8 and 9 it's represented the grid current,  $i_{grid}$ , and the source voltage,  $V_S$ .

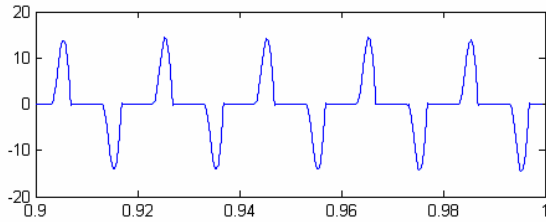


Fig. 8: grid current,  $0.9 \text{ s} < t < 1 \text{ s}$

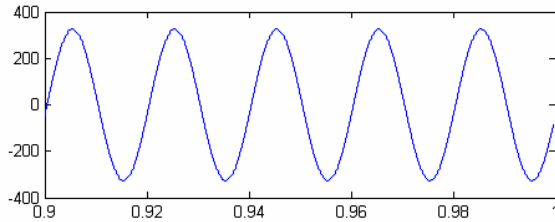


Fig. 9: grid voltage,  $0.9 \text{ s} < t < 1 \text{ s}$

In Figs. 10 and 11, it's represented the harmonic spectre of load current and of grid voltage.

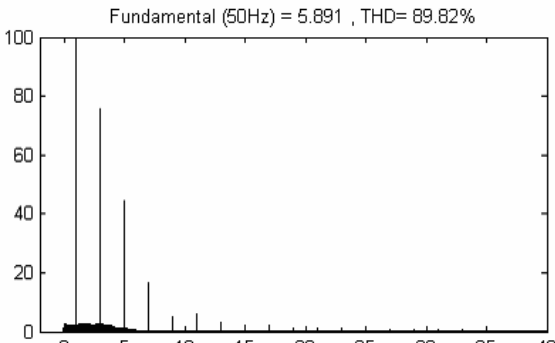


Fig. 10: grid current harmonic spectre

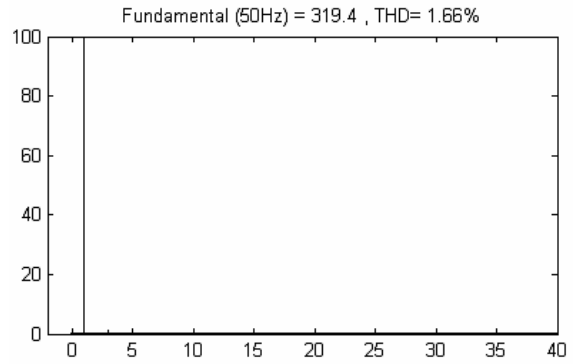


Fig. 11: grid voltage harmonic spectre

Now it's analysed the situation with A.P.F., compensating the load current of Fig. 8. In Figs. 11, 12 and 13 it's represented the grid current, grid voltage and capacitor voltage, respectively.

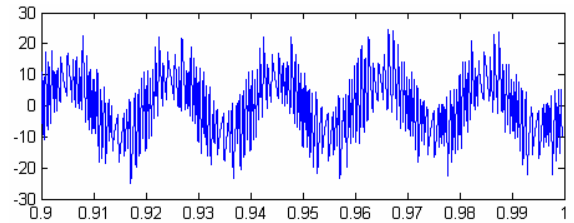


Fig. 11: grid current with A.P.F.,  $0.9 \text{ s} < t < 1 \text{ s}$

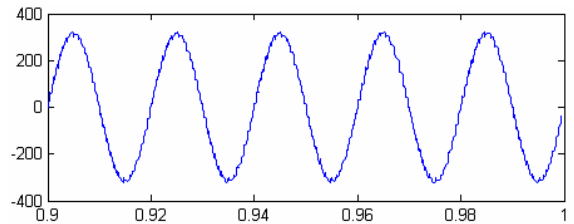


Fig. 12: grid voltage with A.P.F.,  $0.9 \text{ s} < t < 1 \text{ s}$

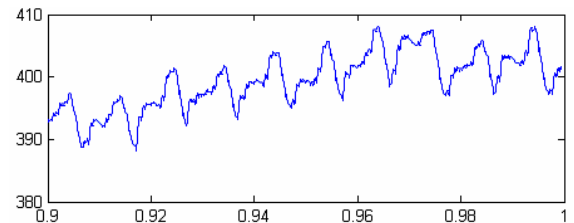


Fig. 13: capacitor voltage,  $0.9 \text{ s} < t < 1 \text{ s}$

In Figs. 14 and 15 it's represented the grid current and grid voltage harmonic spectres.

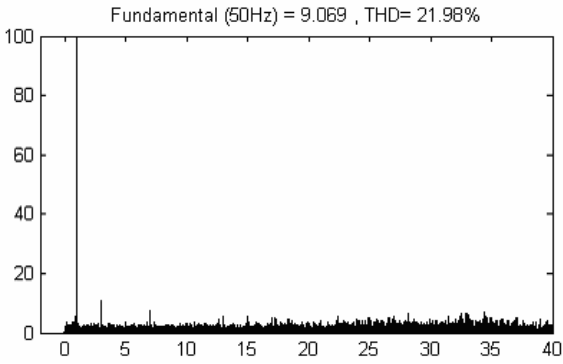


Fig. 14: grid current harmonic spectre, w/ A.P.F.

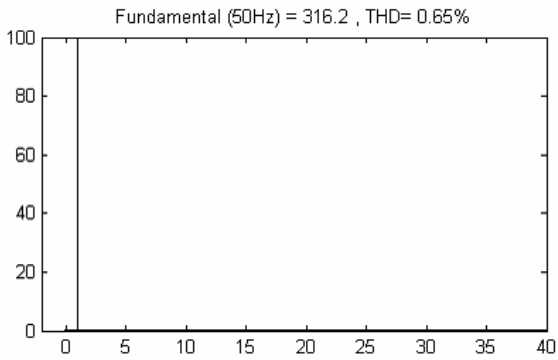


Fig. 15: grid voltage harmonic spectre, with A.P.F.

At last, the system is analysed with A.P.F. commanded by optimal control techniques. Figs. 16, 17 and 18 show the grid current, the grid voltage and the capacitor voltage, respectively.

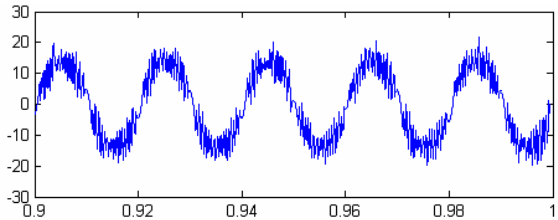


Fig. 16: grid current w/ A.P.F. (optimal control)

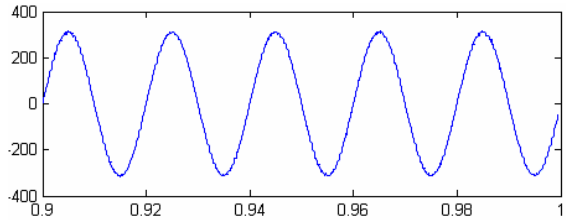


Fig. 17: grid voltage w/ A.P.F. (optimal control)

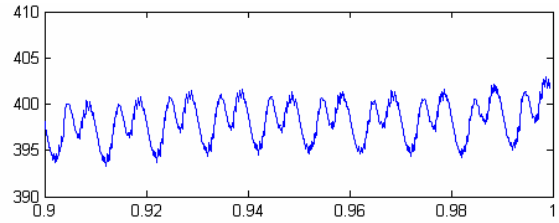


Fig. 18: capacitor voltage w/ A.P.F. (optimal control)

The grid current and grid voltage harmonic spectres are shown in Figs. 19 and 20.

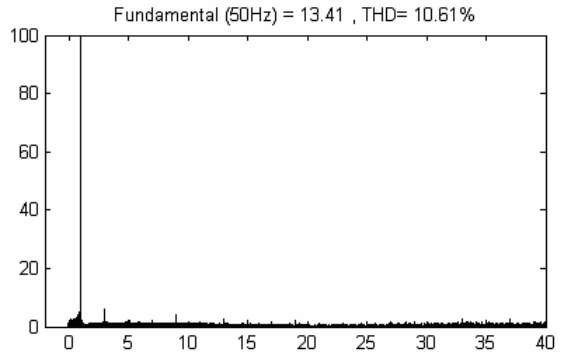


Fig. 19: grid current harmonic spectre

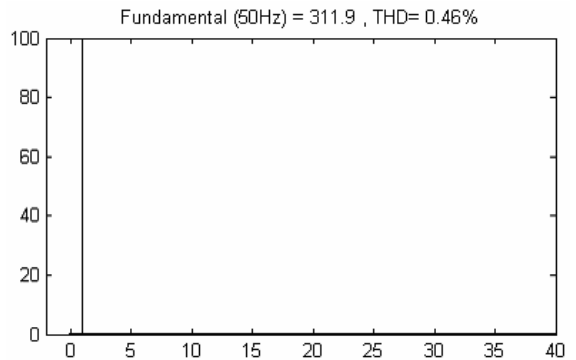


Fig. 20: voltage current harmonic spectre

The results achieved at total harmonic distortions are shown in Table 1.

Table 1: comparison of the THD taxes obtained at three simulations

Ensaio	THDi	THDu	
	Grid current	Grid voltage	Capacitor voltage
No A.P.F.	89,82 %	1,66 %	-
A.P.F. (sliding mode)	21,98 %	0,65 %	0,91 %
A.P.F. (optimal control)	10,61 %	0,46 %	0,69 %

## V. Conclusions

A method based in optimal control to command an active power filter was analysed and proposed in this paper. It was compared with sliding mode control, a conventional method. The results of the introduction of optimal control were compared with other methods. Power systems of the used simulation machine reveal great realism.

The introduction of the A.P.F. allows the THD lowers from nearly 90 % to 21,98 %. With the optimal control method, it's been achieved a reduction of 88,2 %, from 89,92 % to 10,61 %. The introduction of A.P.F. results in great compensation by itself, but with optimal control A.P.F. increases its efficiency dramatically.

## VI. References

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